

# Modified Wilkinson Power Divider for $n$ th Harmonic Suppression

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**Abstract**—This paper presents a structure of the Wilkinson power divider that can suppress the  $n$ th harmonic output. The power divider consists of two  $\lambda/4n$  open stubs, which are located at the center of  $\lambda/4$  branches and a parallel connection of a resistor and an inductor, which shunts the output ports. Experimental results show that this power divider suppresses the third harmonic component to less than  $-40$  dB, while maintaining the characteristics of a conventional Wilkinson power divider; featuring an equal power split, a simultaneous impedance matching at all ports and a good isolation between output ports. These results agree quite well with the simulation results.

**Index Terms**—Harmonic suppression, power combiners, power dividers, power splitter, Wilkinson power combiner.

## I. INTRODUCTION

THE Wilkinson power divider and combiner are being used widely for microwave power amplifiers [1], [2]. Both have the same structure, which consists of two  $\lambda/4$  branches of transmission line and a termination resistor, where  $\lambda$  is the wavelength of the transmission line. They match all input and output ports simultaneously and provide a good isolation between the input ports for the power combiner and between the output ports for the power divider. Also, they can handle arbitrary power levels from input to output ports. If the harmonics are suppressed in the power divider or combiner structure, we can eliminate separate harmonic rejection filters from the circuit and design an area-effective power amplifier.

In this paper, we present a modified Wilkinson power divider that suppresses the  $n$ th harmonic component using two  $\lambda/4n$  open stubs, where  $n$  is the harmonic number that is to be suppressed. By placing a  $\lambda/4n$  open stub at the center of each  $\lambda/4$  branch of the power divider and shunting the output ports with a parallel connection of a resistor and an inductor, the  $n$ th harmonic component and its odd multiples are suppressed without sacrificing the characteristics of the conventional Wilkinson power divider at the operating frequency. In Section II, we propose a structure of the Wilkinson power divider for  $n$ th harmonic suppression. The simulation and experiment results of the Wilkinson power divider for third harmonic suppression are given in Section III and a conclusion is given in Section IV.

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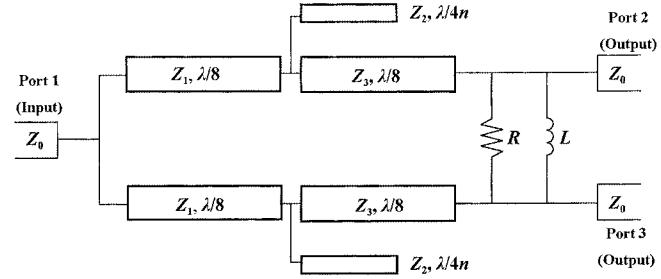


Fig. 1. Schematic diagram of the Wilkinson power divider for  $n$ th harmonic suppression.

## II. WILKINSON POWER DIVIDER FOR $n$ TH ORDER HARMONIC SUPPRESSION

A schematic diagram of the proposed Wilkinson power divider, which suppresses the  $n$ th harmonic component using two  $\lambda/4n$  open stubs, is shown in Fig. 1. The  $\lambda/4$  branch of the original Wilkinson power divider, which has two identical  $\lambda/4$  branches and an output termination resistor, is divided into two  $\lambda/8$  sections. The characteristic impedance of the  $\lambda/8$  section connected to the input port is  $Z_1$  and that connected to the output port is  $Z_3$ , while the input and output impedances are  $Z_0$ . One  $\lambda/4n$  open stub with a characteristic impedance of  $Z_2$  is connected to the junction of the  $\lambda/8$  sections. The output ports are shunted with a parallel connection of a resistor  $R$  and an inductor  $L = X/\omega$ . This power divider is symmetric and we can use the even- and odd-mode analyses to determine the circuit parameters for  $n$ th harmonic suppression [2], [3].

### A. Even-Mode Analysis

In the case where two signals of the same magnitude and phase (even-mode signals) are applied to ports 2 and 3 of the circuit shown in Fig. 1, no current flows through the plane of symmetry. The circuit is bisected at the plane of symmetry for the even-mode analysis. Because no current flows through the plane of symmetry, the circuit elements  $R$  and  $L$ , which shunt the ports 2 and 3, can be eliminated. The impedance  $Z_0$  at port 1 is doubled on the bisected circuit. For  $Z_1 = 2Z_0$ , the impedance for the even mode at port 2,  $Z_{out}^E$ , is calculated using the transmission line impedance equation [2], resulting in

$$Z_{out}^E = Z_3 \frac{4Z_0 Z_2^2 Z_3}{(Z_2 Z_3)^2 + 4Z_0^2 (Z_2 + Z_3 \tan(\frac{\pi}{2n}))^2} + j Z_3 \frac{(Z_2 Z_3)^2 - 4Z_0^2 (Z_2^2 - Z_3^2 \tan^2(\frac{\pi}{2n}))}{(Z_2 Z_3)^2 + 4Z_0^2 (Z_2 + Z_3 \tan(\frac{\pi}{2n}))^2}. \quad (1)$$

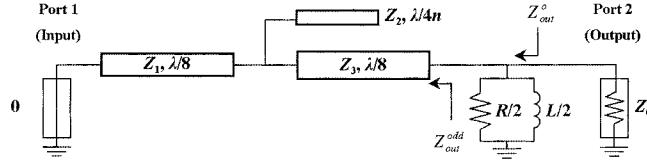


Fig. 2. Circuit of Wilkinson power divider for the odd-mode analysis.

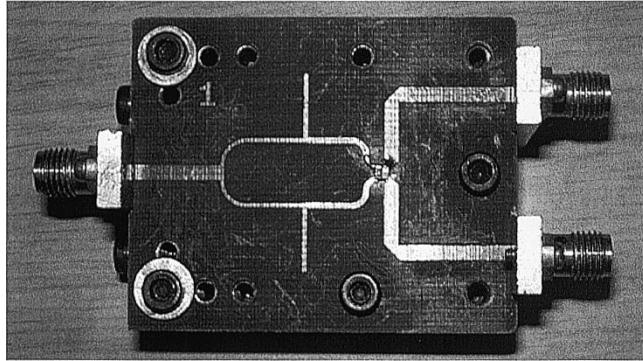


Fig. 3. Photograph of fabricated 2 GHz Wilkinson power divider for third harmonic suppression.

For impedance matching to  $Z_0$ , the imaginary part must be zero, which results in

$$(Z_2 Z_3)^2 - 4Z_0^2 \left( Z_2^2 - Z_3^2 \tan^2 \left( \frac{\pi}{2n} \right) \right) = 0.$$

After defining a positive parameter  $t \equiv Z_3/Z_2$  ( $t > 0$ ), the above equation is solved for  $Z_2$ , resulting in

$$Z_2 = \frac{2Z_0 \sqrt{1 - t^2 \tan^2 \left( \frac{\pi}{2n} \right)}}{t}. \quad (2)$$

Accordingly, the characteristic impedance  $Z_3$  should be

$$Z_3 = 2Z_0 \sqrt{1 - t^2 \tan^2 \left( \frac{\pi}{2n} \right)}. \quad (3)$$

When we insert (2) and (3) into the real part of (1) and enforce the condition for impedance matching  $Z_{out}^E = Z_0$ , the parameter  $t$  satisfies the following equation:

$$2t^2 \tan^2 \left( \frac{\pi}{2n} \right) + t \tan \left( \frac{\pi}{2n} \right) - 1 = 0.$$

The solutions of the above equation are  $t \tan(\pi/2n) = 1/2$  and  $-1$ . Because  $0 < \pi/2n < \pi/2$  and  $t > 0$ , a valid solution is

$$t \tan \left( \frac{\pi}{2n} \right) = \frac{1}{2}.$$

This solution results in

$$Z_2 = 2\sqrt{3} \tan \left( \frac{\pi}{2n} \right) Z_0 \quad (4)$$

$$Z_3 = \sqrt{3} Z_0. \quad (5)$$

For the characteristic impedances  $Z_1 = 2Z_0$ ,  $Z_2 = 2\sqrt{3} \tan(\pi/2n) Z_0$  and  $Z_3 = \sqrt{3} Z_0$ , the impedance for the even mode at port 1 is

$$Z_{in}^E = \frac{Z_2}{j \tan \left( \frac{\pi}{2n} \right)} \parallel Z_3 \frac{Z_0 + Z_3 j \tan \left( \frac{\pi}{4} \right)}{Z_3 + Z_0 j \tan \left( \frac{\pi}{4} \right)} = 2Z_0$$

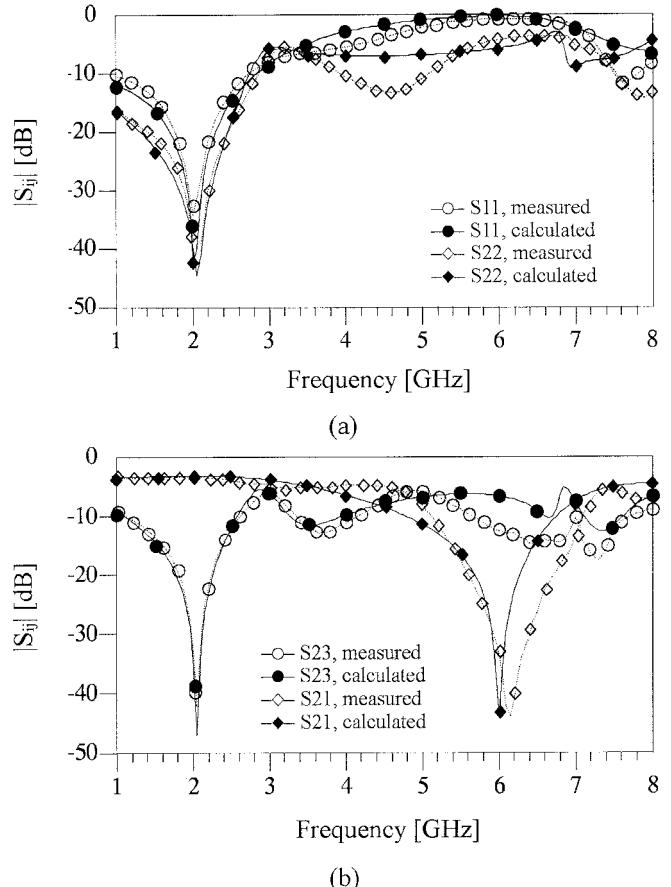


Fig. 4. Measured and calculated  $S$ -parameters of the 2.05 GHz Wilkinson power divider for third harmonic suppression: (a)  $S_{11}$  and  $S_{22}$ . (b)  $S_{23}$  and  $S_{21}$ .

where the symbol  $\parallel$  denotes a parallel connection of circuits. For the circuit shown in Fig. 1, this result gives the input impedance  $Z_{in} = Z_0$ , which is independent of  $n$ . We use this fact to suppress the  $n$ th harmonic component without degrading the properties of the Wilkinson power divider at the fundamental frequency  $f_0$ . At the  $n$ th harmonic frequency  $nf_0$ , the  $\lambda/4n$  open stub shorts the junction of  $\lambda/8$  sections and no transmission from the input to output ports, or vice versa, is possible.

### B. Odd-Mode Analysis

The resistance  $R$  and the inductance  $L$  are determined from the output matching condition. In the case where two signals, which are applied to port 2 and 3 of the circuit shown in Fig. 1, have the same magnitude but  $180^\circ$  out of phase (odd-mode signals), the voltage at the symmetry plane is fixed at 0 V. Accordingly, the circuit can be bisected into the one shown in Fig. 2 for odd-mode analysis. The impedance  $Z_{out}^{odd}$  at the fundamental frequency  $f_0$ , which is the output impedance without  $R$  and  $L$ , is

$$Z_{out}^{odd} = j Z_3 \frac{(Z_1 + Z_3) Z_2 - Z_1 Z_3 \tan \left( \frac{\pi}{4n} \right)}{(Z_3 - Z_1) Z_2 - Z_1 Z_3 \tan \left( \frac{\pi}{4n} \right)} = -j(2 + \sqrt{3}) Z_0.$$

Because the port 2 is shunted with a parallel connection of  $R/2$  and  $L/2$ , the output impedance  $Z_{out}^o$  at port 2 becomes

$$Z_{out}^o = \left( j \frac{1}{(2 + \sqrt{3}) Z_0} + \frac{2}{R} - j \frac{2}{L} \right)^{-1}$$

where  $X \equiv \omega_0 L$  and  $\omega_0 = 2\pi f_0$ . For impedance matching to  $Z_0$  at port 2, we choose

$$R = 2Z_0 \quad (6)$$

and

$$X = 2(2 + \sqrt{3})Z_0. \quad (7)$$

The resistance  $R$  given in (6) is the same as the one for the conventional Wilkinson power divider. For  $Z_0 = 50 \Omega$  and  $f_0 = 2 \text{ GHz}$ , the required  $L$  and  $R$  are  $29.7 \text{ nH}$  and  $100 \Omega$ , respectively.

### III. SIMULATION AND EXPERIMENT

A Wilkinson power divider for  $f_0 = 2 \text{ GHz}$  and  $Z_0 = 50 \Omega$  is fabricated on a 0.78 mm-thick TLC32 substrate from Taconic Ltd., which has a relative permittivity of 3.2 and a conductor thickness of  $35 \mu\text{m}$ . For third harmonic suppression, the power divider has the characteristic impedances of  $Z_1 = 100 \Omega$ ,  $Z_2 = 100 \Omega$  and  $Z_3 = 87 \Omega$ . The resistance  $R$  and inductance  $L$  required for the odd-mode impedance matching are  $100 \Omega$  and  $29.7 \text{ nH}$ . However, the fabricated power divider has an inductance of  $27.0 \text{ nH}$  due to fabrication error. A photograph of the fabricated Wilkinson power divider is shown in Fig. 3. This power divider has an area of  $\sim 3 \times 3 \text{ cm}^2$ . The  $S$ -parameters are measured using an HP8510C network analyzer and are calculated using the ADS software from Agilent Technologies, Inc.

The measured and calculated  $S$ -parameters are shown in Fig. 4. One observes from Fig. 4 that the measured  $S$ -parameters agree quite well with the calculated ones. The measured  $S$ -parameters are shifted from the design goal of  $f_0 = 2 \text{ GHz}$  to  $2.05 \text{ GHz}$  and  $3f_0 = 6 \text{ GHz}$  to  $6.15 \text{ GHz}$ . The sources of this discrepancy include the T-junction effects, the representation errors for simulation and the fabrication errors on  $L$  and  $\lambda/4n$  open stubs. In Fig. 4(a), which shows  $S_{11}$  (○ and ●) and  $S_{22}$  (◇ and ♦), it is shown that the power divider passes the  $2.05 \text{ GHz}$  fundamental signal, but reflects the  $6.15 \text{ GHz}$  third-order harmonic component. At  $2.05 \text{ GHz}$ , the measured  $S_{11}$  and

$S_{22}$  are  $-34 \text{ dB}$  and  $-39 \text{ dB}$ , respectively. In Fig. 4(b), which shows  $S_{23}$  (○ and ●) and  $S_{21}$  (◇ and ♦), it is shown that the  $2.05 \text{ GHz}$  signal at port 1 is equally divided and transmitted to the output ports 2 and 3, but the  $6.15 \text{ GHz}$  third harmonic signal is not transmitted. The measured  $S_{23}$  at  $2.05 \text{ GHz}$  shows that the isolation between ports 2 and 3 is  $\sim 42 \text{ dB}$ . The reflected power at  $2.05 \text{ GHz}$  from one output port has little effect on the operation of the other output port. The measured  $S_{21}$  at  $2.05 \text{ GHz}$  shows a power split of  $\sim 3.3 \text{ dB}$  and that at  $6.15 \text{ GHz}$  shows a third harmonic suppression of  $\sim 44 \text{ dB}$ . These results show that the proposed power divider operates well as a conventional Wilkinson power divider at the operating frequency, while suppressing the third harmonic component.

### IV. CONCLUSION

A structure of the Wilkinson power divider, which can suppress the  $n$ th harmonic output, is proposed. A power divider for  $2.05 \text{ GHz}$  operation has been fabricated and its  $S$ -parameters are measured. At  $2.05 \text{ GHz}$ , the measured  $S_{11}$ ,  $S_{21}$ ,  $S_{22}$  and  $S_{23}$  are  $-34 \text{ dB}$ ,  $-3.3 \text{ dB}$ ,  $-39 \text{ dB}$  and  $-42 \text{ dB}$ , respectively. At the third harmonic frequency of  $6.15 \text{ GHz}$ , the measured  $S_{11}$ ,  $S_{21}$ ,  $S_{22}$  and  $S_{23}$  are  $-0.8 \text{ dB}$ ,  $-44 \text{ dB}$ ,  $-4 \text{ dB}$  and  $-13 \text{ dB}$ , respectively. The results at  $2.05 \text{ GHz}$  show that the power divider has an equal power split to the output ports, good isolation between the output ports and simultaneous impedance matching at all ports. The measured  $S_{21}$  at  $6.15 \text{ GHz}$  shows a third harmonic suppression of  $\sim 44 \text{ dB}$ . These results indicate that the proposed Wilkinson power divider operates well as a conventional Wilkinson power divider at the operating frequency of  $2.05 \text{ GHz}$ , while suppressing the  $6.15 \text{ GHz}$  third harmonic component.

### REFERENCES

- [1] E. Wilkinson, "An N-way hybrid power divider," *IRE Trans. Microwave Theory Tech.*, vol. MTT-8, pp. 116–118, Jan. 1960.
- [2] D. M. Pozar, *Microwave Eng.*: Addison-Wesley, 1990, pp. 395–399.
- [3] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 246–252, Oct. 1956.